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office hour: 9:30 ~ 11:30am. Friday.

HH 401

Preliminary:

Compound Interest: $A(t) = A(0)(1+i)^t$

Simple Interest: $A(t) = A(0)(1+it)$

Present value: $A(0) = A(t)v^t$ $v = \frac{1}{1+i}$ is present value factor

Treasury Bills: price $= \frac{\text{face amount}}{1+it}$

1.1.2.

how to measure

(a) $A(0) = 2500$, $t = \frac{10}{1} = 10$, $i = 4\%$

$A(t) = A(0)(1+it) = 2500 \times (1+4\% \times 10) = 3500.$

(b) $A(t) = A(0)(1+i)^t = 2500 \times (1+4\%)^{10} = 3700.61$

(c) $A(0) = 2500$, $t = \frac{10}{0.5}$, $i = 2\%$,

$A(t) = A(0)(1+i)^t = 2500 \times (1+2\%)^{\frac{10}{0.5}} = 3714.87$

(d) $t = \frac{10}{0.25}$, $i = 1\%$,

$A(t) = A(0)(1+i)^t = 2500 \times (1+1\%)^{\frac{10}{0.25}} = 3722.16.$

1.1.10.

(a) $A(0) = 1000$, $A(t) = 3000$. $i = 12\%$,

$(1+i)^t = \frac{A(t)}{A(0)} \Rightarrow t\log(1+i) = \log A(t) - \log A(0) \Rightarrow t = \frac{\log A(t) - \log A(0)}{\log(1+i)} = 9.694.$

(b) Integer part : 9.

fractional part : 0.6[illegible]

$$A(0)(1+i)^{[t]}(1+iS) = 1000(1+12\%)^9(1+12\% \times S) = 3000 \Rightarrow S = 0.6819$$

$$t = 9 + 0.6819 = 9.6819$$

(c) Let's monthly $i = 1\%$, $1000(1+0.01)^t = 3000 \Rightarrow t = 110.41$ months.

(d) $$(1+i)^t = \frac{A(t)}{A(0)} \Rightarrow i = \sqrt[t]{\left(\frac{A(t)}{A(0)}\right)} - 1 = \sqrt[10]{\frac{3000}{1000}} - 1 = 0.1161$$

(e). $$j = \sqrt[t]{\frac{A(t)}{A(0)}} - 1 = \sqrt[120]{\frac{A(t)}{A(0)}} - 1 = 0.009197$$

1.2.1.

$$v = \frac{1}{1+i} = \frac{1}{1+6\%}$$

present value : $5000v + 5000v^2 + 5000v^3 + 5000v^4 = 17,325.53$

1.2.4.

$$v_{6\%} = \frac{1}{1+6\%}, \quad v_{7\%} = \frac{1}{1+7\%} \quad v_{9\%} = \frac{1}{1+9\%}.$$

$$PV = 1000\, v_{6\%}^3\, v_{7\%}^4\, v_{9\%}^3 = 494.62$$

1.2.15.

(a) $t = \frac{182}{365}$. $\quad P = \frac{\text{face amount}}{1+it} = \frac{100,000}{1+10\% \times \frac{182}{365}} = 95,250.52.$

(b) $P = \frac{F}{1+it}$, $\quad \frac{dP}{di} = -\frac{F}{(1+it)^2} \cdot t$. $\quad \frac{dP}{di} \doteq \frac{P(i+\Delta i) - P(i)}{\Delta i} = \frac{\Delta P}{\Delta i}$, $\Delta i = 10.1\% - 10\%$.

$$\Rightarrow \Delta P = \frac{dP}{di} \cdot \Delta i = -23733.34$$

(c) $t = \frac{91}{365}$ $\quad$ when $t = 0$. $\quad \frac{dP}{di} = -\frac{F}{(1+it)^2} \cdot t = 0.$

1.1.1.

$10{,}000 \times 4\%$, $10{,}000 \times (1.04)^2 - 10{,}000$, $10{,}000 \times (1.04)^3 - 10{,}000$

1.1.3.

$10{,}000(1.01)^3(1.0075)^9 = 10{,}000(1+j)^{12} \Rightarrow j = ?$

1.1.5 (a).

$\sqrt[5]{1.10 \times 1.16 \times 0.93 \times 1.04 \times 1.32} = 1.1025$

1.2.3.

A: 15 16.50

0 2 2

B: 28

$28 = 15 + 16.5v \quad v = \frac{1}{1+i}, \Rightarrow i = ?$

1.2.12.

1000 1092

2000

$1000 + 1092v^2 = 2000v.$

$v = \frac{1}{1+i}. \Rightarrow i = ?$

$1000 + 1092v^2 = 2000v$

$1000 + \frac{1092}{(1+i)^2} = \frac{2000}{1+i} \qquad (1+i)^2$

$1000(1+i)^2 + 1092 = 2000(1+i)$

1.2.13.

(a) $\frac{d}{di}(1+i)^n = n(1+i)^{n-1}$. (b) $\frac{d}{di}\left(\frac{1}{1+i}\right)^n = -n\left(\frac{1}{1+i}\right)^{n+1}$.

(c) $\frac{d}{dn}(1+i)^n = (1+i)^n \ln(1+i)$. (d) $\frac{d}{dn}v^n = -v^n \ln(1+i)$.

1.2.17.

30 30 30 30

0 1 2 ... 23

$30(1+v+\dots+v^{23}) \qquad v = \frac{1}{1+1\%}.$

30 30 30 30

0 1 2 23 24 25

$30(v^2+\dots+v^{25})$

$30(1+v+\dots+v^{23}) - 30(v^2+\dots+v^{25}) = 30(1+v-v^{24}-v^{25})$

$= 17.68.$

$i^{(\infty)} = \lim_{m\to\infty} i^{(m)} = \lim_{m\to\infty} m[(1+i)^{\frac{1}{m}} - 1] = \lim_{m\to\infty} \frac{(1+i)^{\frac{1}{m}}-1}{\frac{1}{m}} = \lim_{x\to 0} \frac{(1+i)^x - 1}{x} = \frac{(1+i)^x \ln(1+i)}{1} = \ln(1+i)$

$i^{(\infty)} = \ln(1+i)$, $f(x) = i^{(m)} = m[(1+i)^{\frac{1}{m}} - 1] = \frac{(1+i)^{\frac{1}{m}}-1}{\frac{1}{m}} = \frac{(1+i)^x - 1}{x}$, $f'(x) = \cancel{\frac{(1+i)^x \ln(1+i) \cdot x - (1+i)^x + 1}{x^2}}$

$\frac{\cancel{m[(1+i)^{\frac{1}{m}} - 1]}}{\ln(1+i)}$

$i^{(m)} = m[(1+i)^{\frac{1}{m}} - 1]$, $i^{(n)} = n[(1+i)^{\frac{1}{n}} - 1]$.

$f'(x)$ $\left(\frac{(1+i)^x - 1}{x}\right)' = \frac{x(1+i)^x \ln(1+i) - (1+i)^x + 1}{x^2}$

$i^{(1)} > i^{(m)} > i^{(\infty)}$. $\qquad (1+i)^x\left(x\ln(1+i) - 1 + \frac{1}{(1+i)^x}\right)$.

$i^{(m)} > i^{(n)}$, $m < n$.

$i^{(m)} - i^{(n)} = m(1+i)^{\frac{1}{m}} - n(1+i)^{\frac{1}{n}} - m + n$.

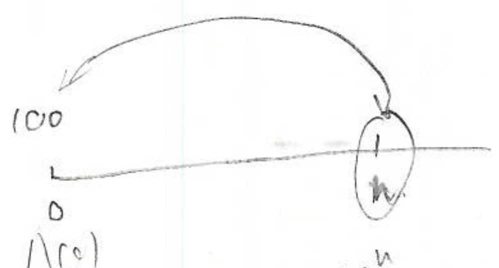
$= \cancel{(\quad)}($

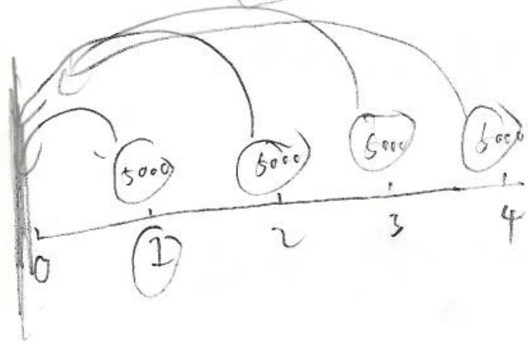
$\frac{100}{1+it}$ $\qquad 100(1-dt) = 100\left(1 - \frac{i}{1+i}t\right)$

$= 100\left(\frac{1+i-it}{1+i}\right)$

$\frac{100}{1+\frac{d}{1-d}t} = \frac{100(1-d)}{1-d+dt}$

$5000v + 5000v^2 + 5000v^3 + 5000v^4 =$





$100(1+i)^n$

$A(t) = A(0)(1+i)^t$.

$A(0) = \frac{A(t)}{(1+i)^t} \Rightarrow A(0) = A(t)v^t$

$v = \frac{1}{1+i}$,

1.1.2

(a) $2500 \times (1+4\% \times 10) = 3500$ $\quad C(1+in)$ $\quad n = \frac{T}{t}$

(b) $2500 \times (1+4\%)^{10} = 3700.61$ $\quad C(1+i)^n$

(c) $2500 \times (1+2\%)^{\frac{10}{0.5}} = 3714.87$

(d) $2500 \times (1+1\%)^{\frac{10}{0.25}} = 3722.16$

1.1.10.

(a) $A(t) = A(0)(1+i)^t$ $\quad 3000 = 1000(1+12\%)^t \Rightarrow t = 9.694$

(b) $A(t) = A(0)(1+it)$ $\quad 1000(1+12\%)^9 \cdot (1+12\% \times s) = 3000 \Rightarrow s = 0.6819$

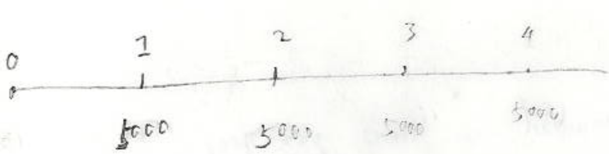
$t = 0.6819 + 9$

(c) $1000(1.01)^t = 3000 \Rightarrow t = 110.41$ months

(d) $1000(1+i)^{10} = 3000$ $\quad i = 0.1161$

(e) $1000(1+j)^{10\times 12} = 3000$ $\quad i = 0.009197$

1.2.1. $5000 \times \frac{1}{1+6\%} + 5000 \times \frac{1}{(1+6\%)^2} + 5000 \times \frac{1}{(1+6\%)^3} + 5000 \times \frac{1}{(1+6\%)^4} = 17,325.53$



1.2.4

$X(1+6\%)^3(1+7\%)^4(1+9\%)^3 = 1000$

$X = 1000\, v_{.06}^3\, v_{.07}^4\, v_{.09}^3 = 499.62$

v: present value factor

1.2.15. (Pricing T-bill).

(a) $\dfrac{100,000}{1+10\% \times \frac{182}{365}} = 95,250.52.$ $\quad$ price $= \dfrac{\text{face amount}}{1+it}$

(b) $P = \dfrac{F}{1+it}$ $\quad \dfrac{dP}{di} = d\left(\dfrac{F}{1+it}\right)/di = -\dfrac{F}{(1+it)^2}\cdot t$

$P(i)$

$\dfrac{dP}{di} \doteq \dfrac{P(i+\Delta i) - P(i)}{\Delta i}$

$\Delta P = -\dfrac{100,000}{(1+i\frac{182}{365})^2}\cdot \dfrac{182}{365} * \Delta i = -45.24$

$i = 0.1$

(c) $\dfrac{dP}{di} = -\dfrac{1000}{(1+i\frac{91}{365})^2}\cdot\dfrac{91}{365} = -23,733.34 \quad (i = 0.10).$

$t = 0.$

Compound Interest: $A(0)(1+i)^t = A(t)$

simple Interest: $A(0)(1+it) = A(t)$ (For T-bill).

Present value: $A(0) = \dfrac{A(t)}{(1+i)^t} = A(t)v^t.$ $\quad v = \frac{1}{1+i}$, discount factor.

T-bill $\quad$ price $= \dfrac{\text{face amount}}{1+it}$

$i^{(m)}$ nominal annual rate with interest compounded m times per year.

i annual rate $\quad 1+i = \left[1+\dfrac{i^{(m)}}{m}\right]^m.$ $\quad i^{(m)} = m\left[(1+i)^{\frac{1}{m}} - 1\right].$

$i^{(\infty)} = \ln(1+i).$ $\quad 1+i = e^{i^{(\infty)}}.$